On the compilation of bottom topographic detail for the Bay of Bengal region through inverse distance weighted interpolation

Md. Mamunur Rasid, Md. Morshed Bin Shiraj, Md. Dalim Haque, Md. Masum Murshed, Md. Rabiul Haque

Abstract— In this study, the bottom topographic detail for the Bay of Bengal region covering the area between 15^0N and 23^0N Latitudes and 85^0E and 95^0E Longitudes is compiled through inverse distance weighted interpolation from a figure with some doted depth contours. To ensure the depth values at every consecutive column in the required matrix of depths, the doted depth contours are made continuous using the cubic spline interpolation. From the figure with continuous depth contours, a matrix of depth values is constructed by a MATLAB routine. Then inverse distance weighted interpolation is used to generate depth values at all other elements of the matrix. Such a type of depth matrix of the Bay of Bengal region can be used to compile the bathymetric data of that region properly.

Keywords — Bottom topography, MATLAB, cubic spline interpolation, inverse distance weighted interpolation, Bay of Bengal.

1 INTRODUCTION

DEPTH at a point on the sea in the mean sea level is the distance of the sea bed from that point. The collection of depths of all points of a region of the sea is called the bottom topographic detail of that region. It is of interest to note here that the tropical storms along with associated tidal surges often cause great devastation along the coast of Bangladesh (see [3]). According to Paul et al. [5, 6], on an average 5-6 storms form in this region every year causing 80% of global casualties. The coastal belt of Bangladesh, especially the Meghna estuarine area, is the most vulnerable and the major factors behind the vulnerability can be found in [4]. But the area is insufficiently studied (see [5]). Therefore, an efficient surge forecasting model for the coast of Bangladesh is highly desirable, which in turns can minimize the resulting damages. Though this region is the world's most vulnerable one, but the area is insufficiently studied. Thus the coastal region of Bangladesh is chosen as our study area. It is to be noted here that the bottom topographic detail is an initial input in developing a storm surge prediction model in any region. Also, in shallow water, surge height is very sensitive to the depth of the ocean (see [2]). Therefore, the bathymetric data should properly be incorporated for the accurate prediction of water levels due to a storm along the coast of Bangladesh.

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In this study, we intend to compile the bottom topographic data for the Bay of Bengal region through IDWI from a picture used in the study of Johns et al. [1] (see Fig. 1) in a very efficient way, which in turns may help to develop an efficient storm surge prediction model for the coast of Bangladesh.

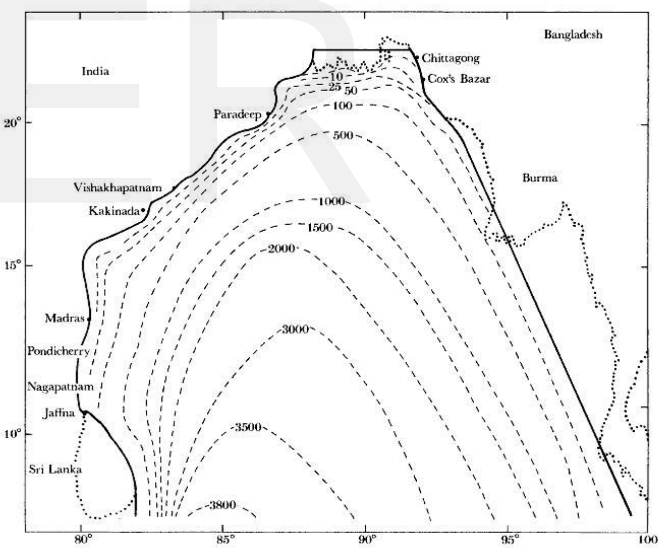

Fig. 1 Bathymetric figure after Johns et al. [1] from which water depth data are compiled

2 NECESSARY BACKGROUND

2.1 Cubic spline interpolation

Suppose that we are given $x_0 < x_1 < x_2 < \dots\dots < x_n$ with the corresponding values $y_0, y_1, y_2, \dots\dots, y_n$, respectively. Then the cubic spline is defined by,

$F(x) = F_i(x)$, for $x_i \le x \le x_{i+1}$, where $F_i(x) = p_i x^3 + q_i x^2 + r_i x + s_i$, for $i = 0,1,2,\dots\dots\dots,n-1$ and each of p_i, q_i, r_i, s_i is real constant (unknown).

Therefore, the total number of unknowns is $4n$. The conditions of the cubic spline are that the functions $F(x)$, $F'(x)$ and

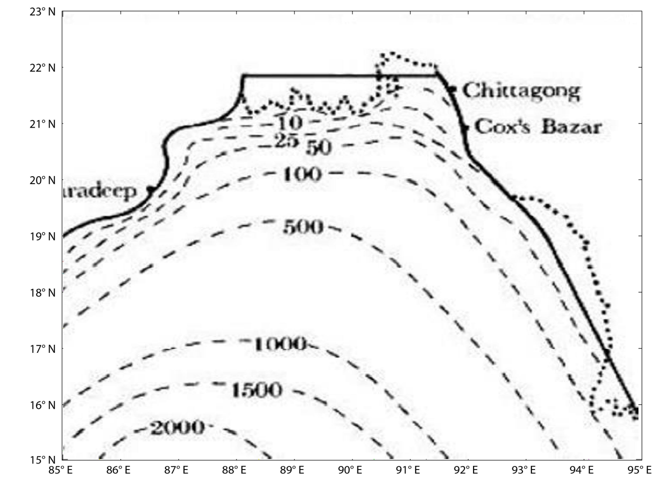

Fig. 2 Bathymetric figure covering the area between 15^0N and 23^0N Latitudes and 85^0E and 95^0E Longitudes taken from Fig. 1

$F''(x)$ are all continuous and $F(x)$ satisfies all the given data points. These conditions give the following $4n-2$ equations:

(i) $F_i(x_i) = y_i$, for $i = 0,1,2,......,n-1$; (n equations)

(ii) $F_i(x_{i+1}) = y_{i+1}$, for $i = 0,1,2,......,n-1$; (n equations)

(iii) $F_i'(x_{i+1}) = F_{i+1}'(x_{i+1})$, for $i = 0,1,2,......,n-2$; ($n-1$ equations)

(iv) $F_i''(x_{i+1}) = F_{i+1}''(x_{i+1})$, for $i = 0,1,2,......,n-2$; ($n-1$ equations)

For the natural cubic spline we have the following two more conditions:

(v) $F_0''(x_0) = F_{n-1}''(x_n) = 0$ (2 equations)

Thus, we have total $4n$ equations with $4n$ unknowns. Since each $F_i(x)$ is of third degree, each $F_i'(x)$ is of second degree and each $F_i''(x)$ is of first degree polynomial. As $F_i''(x)$ is linear, we can use Lagrange form of $F_i''(x)$. Then integrating $F_i''(x)$ twice, we get $F_i(x)$. Integrating constants can be computed using the conditions (i)-(v). Let us define $w_i = F_i''(x_i)$, for $i = 1,2,3,......,n-1$. Then (v) gives $w_0 = w_n = 0$. Now, the rest of w_i's are unknowns. Setting $k_i = x_{i+1} - x_i$, Lagrange form of $F_i''(x)$ becomes

$$F_i''(x) = \frac{w_{i+1}}{k_i}\left(x - x_i\right) - \frac{w_i}{k_i}\left(x - x_{i+1}\right).$$

Then integrating this with respect to x, we have

$$F_i'(x) = \frac{w_{i+1}}{2k_i}\left(x - x_i\right)^2 - \frac{w_i}{2k_i}\left(x - x_{i+1}\right)^2 + t_i - u_i \text{ .... (A)}$$

Again, integrating (A) with respect to x, we have

$$F_i(x) = \frac{w_{i+1}}{6k_i}\left(x - x_i\right)^3 - \frac{w_i}{6k_i}\left(x - x_{i+1}\right)^3 \text{ .... (B)}$$

$$+t_i\left(x - x_i\right) - u_i\left(x - x_{i+1}\right),$$

where t_i and u_i are constants.

Now, using equations (i) and (ii), we have

$$u_i = \frac{y_i}{k_i} - \frac{k_i w_i}{6} \text{ and } t_i = \frac{y_{i+1}}{k_i} - \frac{k_i w_{i+1}}{6}.$$

Also, from the equations (A) and (B), we have

$$F_i'(x) = \frac{w_{i+1}}{2k_i}(x - x_i)^2 - \frac{w_i}{2k_i}(x - x_{i+1})^2 + \frac{y_{i+1}}{k_i} - \frac{k_i w_{i+1}}{6} - \frac{y_i}{k_i} + \frac{k_i w_i}{6}$$

Later, using the continuity of $F'(x)$, i.e., $F_{i-1}'\left(x_i\right) = F_i'\left(x_i\right)$, for $i = 1,2,......,n-1$, we get

$$F_i'\left(x_i\right) = -\frac{k_i w_{i+1}}{6} - \frac{k_i w_i}{3} + m_i, \text{ where } m_i = \frac{y_{i+1} - y_i}{k_i}.$$

From which, we have

$$F_{i-1}'\left(x\right) = \frac{k_{i-1}w_{i-1}}{6} + \frac{k_{i-1}w_i}{3} + m_{i-1}.$$

Finally, we get

$k_{i-1}w_{i-1} + 2\left(k_{i-1} + k_i\right)w_i + k_i w_{i+1} = 6\left(m_i - m_{i-1}\right)$, where $i = 1,2,......,n-1$ and $w_0 = w_n = 0$.

This can be written in matrix form as $H \cdot \overline{w} = \overline{m}$, where

$$H = \begin{pmatrix} 2(k_0 + k_1) & k_1 & & & & \\ k_1 & 2(k_1 + k_2) & k_2 & & & \\ & k_2 & 2(k_2 + k_3) & k_3 & & \\ & & \cdots & \cdots\ \cdots & & \\ & & & \cdots\ \cdots\ \cdots & & \\ & & & \cdots\ \cdots & \cdots & \\ & & & k_{n-3} & 2(k_{n-3} + k_{n-2}) & k_{n-2} \\ & & & & k_{n-2} & 2(k_{n-2} + k_{n-1}) \end{pmatrix},$$

$$\overline{w} = \begin{pmatrix} w_1 \\ w_2 \\ w_3 \\ \cdot \\ \cdot \\ \cdot \\ w_{n-2} \\ w_{n-1} \end{pmatrix} \text{ and } \overline{m} = \begin{pmatrix} 6(m_1 - m_0) \\ 6(m_2 - m_1) \\ 6(m_3 - m_2) \\ \cdot \\ \cdot \\ \cdot \\ 6(m_{n-2} - m_{n-3}) \\ 6(m_{n-1} - m_{n-2}) \end{pmatrix}$$

From the above, we observe that H is a tri-diagonal symmetric and diagonal dominant, that is, $2\left|k_{i-1}+k_i\right| > \left|k_i\right| + \left|k_{i-1}\right|$ which imply unique solution of the system. Hence, solving the system for w_i's, each of $F_i\left(x\right)$ can be computed using them.

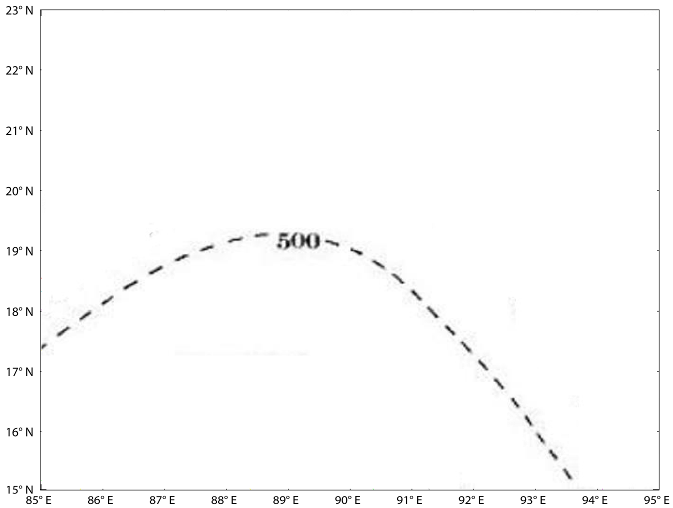

Fig. 3 A doted depth contour taken from Fig. 2

2.2 Inverse distance weighted interpolation (IDWI)

Inverse distance weighted Interpolation is a type of deterministic method for multivariate interpolation with a known scattered set of points. The assigned values to unknown points are calculated with a weighted average of the values available at the known points. The set of known n data points can be described as a list of tuples: $\left[\left(x_1,u_1\right),\left(x_2,u_2\right),......,\left(x_n,u_n\right)\right]$.

A general form of finding an interpolated value at a given point based on samples $u_i\left(x\right)=u_i$, for $i=1,2,3,......,n$, for using IDWI is an interpolating function:

$$u(x)=\begin{cases}\dfrac{\sum_{i=1}^{n} w_i(x)u_i}{\sum_{i=1}^{n} w_i(x)} & if \quad d(x,x_i)\neq 0 \quad for \quad all \quad i \\ u_i & if \quad d(x,x_i)=0 \quad for \quad some \quad i\end{cases}$$

where $w_i\left(x\right)=\dfrac{1}{d\left(x,x_i\right)^p}$ is a simple IDWI weighting function, x denotes an interpolated (arbitrary) point, x_i is an interpolating (known) point, d is a given distance (metric operator) from the known point x_i to the unknown point x, n is the total number of known points used in the interpolation and p is a positive real number, called the power parameter.

3 Methodology

First of all, a picture covering the area between Latitudes 15⁰N and 23⁰N and Longitudes 85⁰E and 95⁰E (see Fig. 2) is taken from the Fig. 1 using the *Microsoft Office* software. Then a single contour is considered and all other contours are omitted from that picture using the *Paint* software (see Fig. 3). Then Fig. 3 is converted into matrix of pixels using a MATLAB routine. As the image (Fig. 3) is considered in RGB, each pixel has three components, first one for contribution of red, second one for contribution of blue and third one for contribution of green. Setting the first column of pixels as y-axis and the last row as x-axis, the data of the contour is extracted from the cilour information of that contour. Then using the natural cubic spline interpolation (see subsection 2.1), the doted contour is made continuous (see Fig. 4). Then by a MATLAB routine, a matrix of that picture (Fig. 4) is generated and the depth information of that contour are extracted to another matrix of the same dimension setting all other depths to be zero. In the similar way, all the doted contours are made continuous (see Fig. 5) and different depth matrices are generated for each of

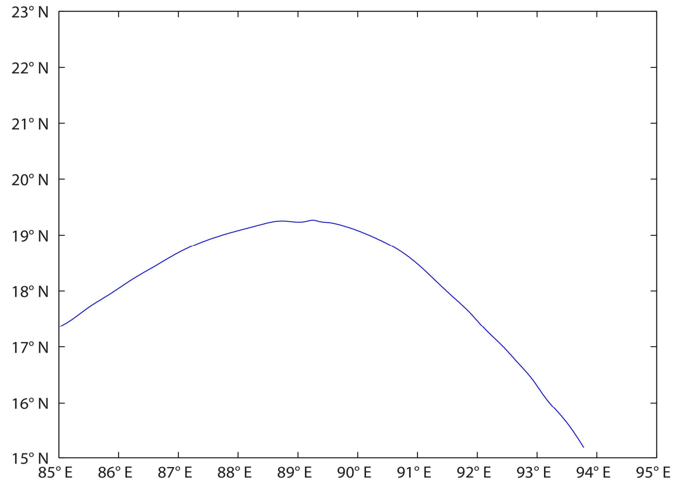

Fig. 4 A continuous depth contour made from Fig. 3 using cubic spline

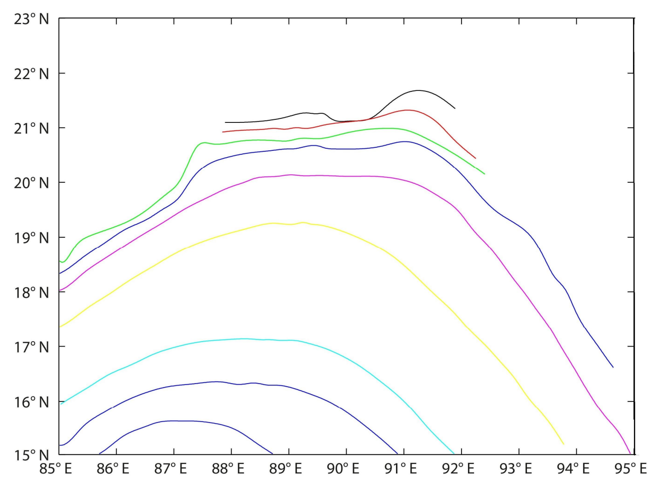

Fig. 5 Continuous depth contours made from Fig. 2 using cubic spline interpolation

the other contours. Then by simple addition of matrices all the depth information are taken in a single matrix (here after will be called the depth matrix). Subsequently, water and land area are identified from Fig. 2 using a MATLAB routine and presented in two different colours (see Fig. 6). Then using a MATLAB routine, another matrix of the same dimension is

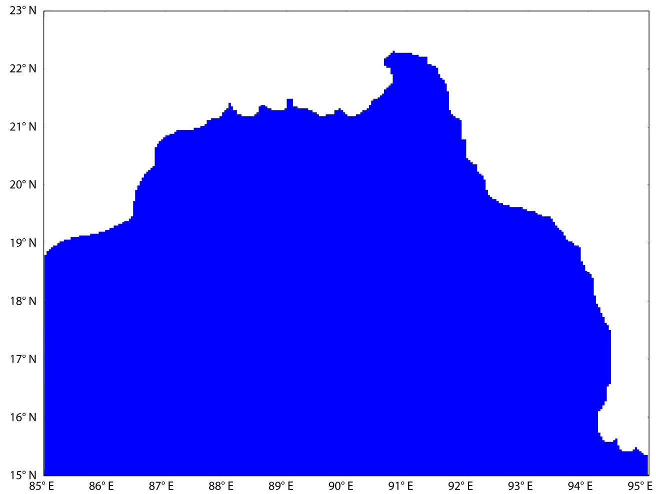

Fig. 6 Water and land area of Fig. 2, here blue colour represents water and white colour represents land

generated from the image presented in Fig. 6 consisting of 1 and 2 representing water and land, respectively. Finally, IDWI (see subsection 2.2) is used to generate the depths at those elements of the depth matrix which are zero but in the water region. The value of the power parameter is taken as 2 and the Euclidean metric is taken as the metric operator. A contour map of our interpolated bottom topography is shown in Fig. 7.

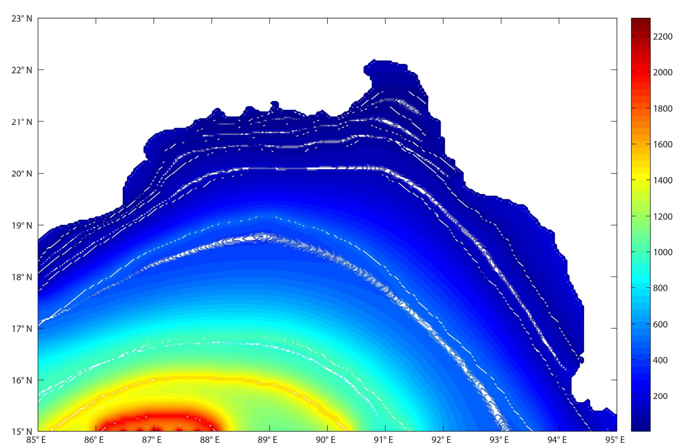

Fig. 7 A contour map of our interpolated bathymetry using inverse distance weighted interpolation

4 Discussion of Results and Conclusion

An accurate bathymetric data of the Bay of Bengal region covering the area between 15^0–23^0N Latitudes and 85^0–95^0E Longitudes are compiled accurately in a matrix using IDWI from a picture of doted contours (see Fig. 2). The obtained bathymetric data can be used to solve shallow water equations for the prediction of water levels due to surge, tide or their interaction and many other purposes (such as safety of surface, subsurface navigation, detecting something, submarine engineering etcetera) on this region. The method of compilation used in this study can be applied to compile the bathymetric data for any coastal region having a figure of doted depth contours.

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Ethical Statement

The authors declare that they have no conflict of interest.

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